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Improved Fourier Transform for Processes with Initial Cyclic-Steady-State

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A new process identification method is proposed to estimate the frequency responses of the process from the activated process input and output. It can extract many more frequency responses and guarantees better accuracy than the previous describing function analysis algorithm. In addition, the proposed method can be applied to the case that the initial part of the activated process data is periodic (cyclic-steady-state), which is not possible with any previous nonparametric identification methods using the modified Fourier transform or Fourier analysis. Furthermore, it can incorporate all the cases in which either the initial part is steady-state and the final part is cyclic-steady-state or both the initial and final parts are steady-state. *AIChE J*, 56: 1536–1544, 2010
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Introduction

The majority of the controllers used in industry are PID controllers due to satisfactory control performance for usual processes and robustness to modeling errors. The tuning parameters of the PID controller have to be tuned with in-depth consideration of the process dynamics to guarantee acceptable control performances. Since the manual tuning depends on the experience, it is inefficient and time-consuming. Many studies, therefore, have focused on the autotuning of the PID controller.

Åström and Hägglund[1] proposed the original relay feedback identification method for the automatic tuning of PID controllers. It used the describing function analysis to obtain an approximated critical point from the relay oscillation. Their idea has been applied in many areas.[2–7] The describing function analysis has been widely used to identify the ultimate information from the relay feedback signal.[1] It is derived using the Fourier series of the relay feedback signal, where only the fundamental term of the series is considered. In general, the ultimate frequency and gain estimated by the describing function analysis have acceptable accuracy for usual processes.[8]

However, since the square signal is approximated by one sinusoidal signal, the harmonic terms could be dominant. Sung et al. proposed a modified relay feedback method to obtain the ultimate data set more accurately.[9] Here, they used a two-level signal instead of the one-level signal of the original relay feedback to reduce the harmonic terms. Also, Shen et al. used a saturation-relay feedback method to reduce the harmonic terms.[10] By applying the describing function analysis to the integrals of the relay feedback signal, Lee et al. gained a significant reduction of the harmonics.[11] Although the aforementioned approaches have contributed to improving the accuracy of the estimates, the estimation errors originating from the describing function approximation still remain.

Sung and Lee proposed the Fourier analysis method to overcome the problems of the describing function analysis method.[12] Despite its ability to estimate the exact frequency response data of the process without any approximations, it

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can provide only one or two frequency response data, because it uses only the process data of the cyclic-steady-state.

Luyben proposed an identification method for the case of steady-state using the Fourier transform.[13] Although it can provide exact frequency models for a wide range of frequencies, it is valid only for the case of both the initial and final steady-state.

Sung and Lee,[14] Ma and Zhu,[15] and Park et al.[16] proposed the most advanced nonparametric identification algorithm using a modified Fourier transform to provide many more frequency response data than the previous describing function analysis algorithm[1] and the Fourier analysis algorithm.[12] Furthermore, the estimates are exact.

However, all the previous methods using the modified Fourier transform can be applied only to the cases that the initial and final parts of the activated process data are steady-state and cyclic-steady-state, respectively. They cannot incorporate the cases in which both the initial and final parts are cyclic-steady-state. Therefore, we here propose a new frequency model identification method to overcome the limitations of the previous approaches. The proposed method can incorporate all the aforementioned cases and theoretically provides the exact frequency responses of the process.

Process Activation

The proportional controller and relay feedback method have been widely used to activate the process due to their simple implementations and ability to provide important dynamic information of the process with minimally perturbing the process. This research considers three types of the process activation by the proportional controller and relay feedback methods as shown in Figure 1. Both the initial and the final parts of Figure 1a are steady-state. In this case, the proportional controller is used to activate the process. In Figure 1b, the process is activated by a biased-relay feedback method. The notable feature of this case is that the initial part of the signal is steady-state and the final part is cyclic-steady-state. In Figure 1c, the conventional biased-relay feedback method is used to activate the process. The initial unsteady-state response of the process is stabilized to obtain the initial cyclic-steady-state. The reference value for the relay on-off is changed at $t = 14$.

Previous Identification Methods to Obtain Frequency Responses

Several methods such as the describing function analysis,[1] Fourier analysis,[12] Fourier transform for steady-state,[13] and modified Fourier transform[14–15] have been proposed to identify the frequency model of the process. The describing function analysis is applicable to Figure 1b and 1c. This method can obtain only one or two frequency responses and generates significant modeling errors, originating from the harmonics, in estimating frequency responses. The Fourier analysis can identify the exact frequency responses of the process in the case of Figure 1b and c. However, it can obtain only one or two frequency response data. The Fourier transform for steady-state can be applied to the case of Figure 1a. Despite providing a wide range of frequency information and guaranteeing reliable accuracy, it cannot incorporate the other cases of Figure 1b and c. The modified Fourier transform can estimate theoretically the complete range of the frequency responses in the case of Figure 1b and it provides the exact estimates. However, it cannot be applied to the cases of Figure 1c. Table 1 summarizes the previous approaches, none of which can be applied to all the cases of Figure 1 to estimate the complete range of the frequency responses of the process with reliable accuracy.

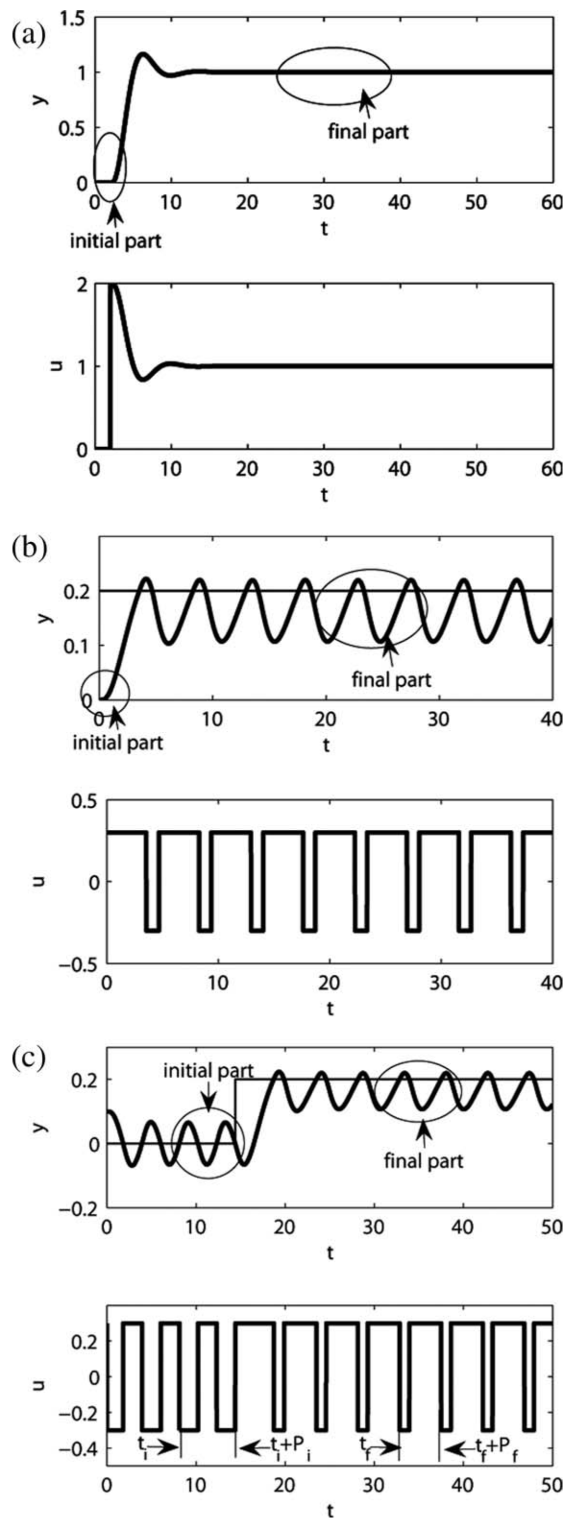

Figure 1. Three cases of process activation.

Table 1. Summary of Previous Approaches

Applications	Algorithms			
	DFA	FA	FTS	MFT
Initial steady-state and final steady-state	Not applicable	Not applicable	Applicable	Not applicable
Initial steady-state and final cyclic-steady-state	Applicable	Applicable	Not applicable	Applicable
Initial cyclic-steady-state and final cyclic-steady-state	Applicable	Applicable	Not applicable	Not applicable
Number of estimated frequency responses	Only one or two	Only one or two	Theoretically all	Theoretically all
Accuracy	Approximated	Exact	Exact	Exact

DFA: Describing Function Analysis, FA: Fourier Analysis, FTS: Fourier Transform for Steady-State, MFT: Modified Fourier Transform.

In this study, a new nonparametric process identification algorithm, capable of estimating a wide range of frequency responses for all the cases of Figure 1, is proposed to overcome the limitations of the previous approaches.

Proposed Identification Method

Consider the activated process input and output of Figure 1c, where $u(t)$ and $y(t)$ denote the process input and output, respectively. In this section, an improved Fourier transform will be developed to estimate the frequency responses from the process input and output in Figure 1c.

The following assumptions and definitions are used in developing the proposed method.

Assumption 1: The initial part from t_i to $t_i + P_i$, and the final part from t_f to $t_f + P_f$ of the activated process input and output are cyclic-steady-state.

Assumption 2: The dynamics of the process is described by the following linear time-invariant transfer function

$$G(s) = \frac{b_m s^m + b_{m-1}s^{m-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + 1} \quad (1)$$

This is equivalent to the following differential equation

$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}} + \cdots + a_1 \frac{dy(t)}{at} + y(t) = b_m \frac{d^m u(t)}{dt^m} + b_{m-1}\frac{d^{m-1}u(t)}{dt^{m-1}} + \cdots + b_1 \frac{du(t)}{dt} + b_0 u(t) \quad (2)$$

Definition 1: Let the following transform for a signal of $\bar{y}(t)$ be defined as

$$\bar{y}_t(s) = \int_0^t e^{-s\tau}\bar{y}(\tau)d\tau \quad (3)$$

If the initial part of the signal is zero-steady-state, then the following property of the transform is easily proven

$$\left\{\frac{d^n\bar{y}}{dt^n}\right\}_t(s) = \int_0^t e^{-st}\frac{d^n\bar{y}(t)}{dt^n}dt = s\left\{\frac{d^{n-1}\bar{y}}{dt^{n-1}}\right\}_t(s) + e^{-st}\frac{d^{n-1}\bar{y}(t)}{dt^{n-1}} \quad (4)$$

Data processing

Consider $u_{ref}(t)$ and $y_{ref}(t)$ in Figure 2. The signals are obtained by repeating the initial part from t_i to $t_i + P_i$ of Figure 1c. Next, define the deviation variables of $\bar{u}(t) = u(t) - u_{ref}(t)$ and $\bar{y}(t) = y(t) - y_{ref}(t)$. Then, the initial part of $\bar{u}(t)$ and $\bar{y}(t)$ from t_i to $t_i + P_i$ is zero-steady-state, and the final part of $\bar{u}(t)$ and $\bar{y}(t)$ after t_f is a cyclic-steady-state of which the period is the common multiple of P_i and P_f ($P_r = mP_i = nP_f$, m and n are integers). The proposed algorithm developed below estimates the frequency responses from the deviation variables.

Algorithm 1

Theoretically, Algorithm 1 developed in this subsection can estimate the complete range of the frequency responses from the deviation variables. Nevertheless, it cannot be applied to real situations because of the heavy computation load. So, it will, therefore, be modified to Algorithm 2 for practical use in the next subsection.

Equation 2 is valid for $u_{ref}(t)$ and $y_{ref}(t)$ of Figure 2. So

$$a_n \frac{d^n y_{ref}(t)}{dt^n} + a_{n-1}\frac{d^{n-1}y_{ref}(t)}{dt^{n-1}} + \cdots + a_1 \frac{dy_{ref}(t)}{at} + y_{ref}(t) = b_m \frac{d^m u_{ref}(t)}{dt^m} + b_{m-1}\frac{d^{m-1}u_{ref}(t)}{dt^{m-1}} + \cdots + b_1 \frac{du_{ref}(t)}{dt} + b_0 u_{ref}(t) \quad (5)$$

By subtracting Eq. 5 from Eq. 2, Eq. 6 is obtained

$$a_n \frac{d^n \bar{y}(t)}{dt^n} + a_{n-1}\frac{d^{n-1}\bar{y}(t)}{dt^{n-1}} + \cdots + a_1 \frac{d\bar{y}(t)}{at} + \bar{y}(t) = b_m \frac{d^m \bar{u}(t)}{dt^m} + b_{m-1}\frac{d^{m-1}\bar{u}(t)}{dt^{m-1}} + \cdots + b_1 \frac{d\bar{u}(t)}{dt} + b_0 \bar{u}(t) \quad (6)$$

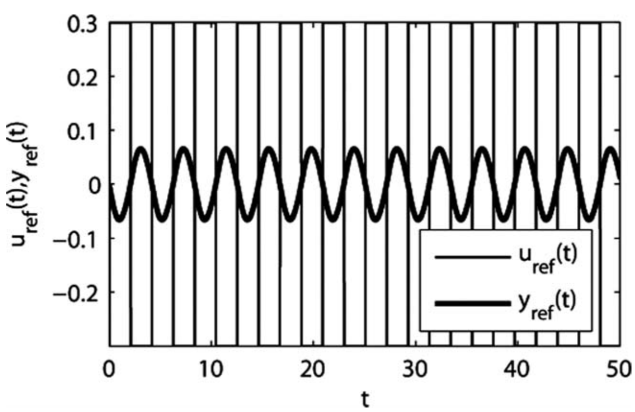

Figure 2. Periodic reference signal obtained by repeating the initial part from t_i to $t_i + p_i$.

Equations 7–9 are obtained by applying Eqs. 3 and 4 to Eq. 6

$$\begin{aligned}&den(s)\bar{y}_t(s) + e^{-st}\\ &\times\left\{\frac{(den(s)-1)}{s}\bar{y}(t) + A_1(s)\frac{d\bar{y}(t)}{dt} + \cdots + A_{n-1}(s)\frac{d^{n-1}\bar{y}(t)}{dt^{n-1}}\right\}\\ &= num(s)\bar{u}_t(s) + e^{-st}\\ &\left\{\frac{(num(s)-b_0)}{s}\bar{u}(t) + B_1(s)\frac{d\bar{u}(t)}{dt} + \cdots + B_{m-1}(s)\frac{d^{m-1}\bar{u}(t)}{dt^{m-1}}\right\}\end{aligned} \quad (7)$$

$$den(s) = a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + 1 \quad (8)$$

$$num(s) = b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0 \quad (9)$$

Here, A_1 (s), …, $A_{n-1}(s)$ and B_1 (s), …, $B_{m-1}(s)$ are time-independent constants. Then, Eqs. 7–9 can be rewritten as

$$\begin{aligned}&s \times den(s)\bar{y}_t(s)e^{st} + den(s)\bar{y}(t) - \bar{y}(t) + sA(s,t)\\ &= s \times num(s)\bar{u}_t(s)e^{st} + num(s)\bar{u}(t) - b_0\bar{u}(t) + sB(s,t)\end{aligned} \quad (10)$$

$$A(s,t) = A_1(s)\frac{d\bar{y}(t)}{dt} + \cdots + A_{n-1}\frac{d^{n-1}\bar{y}(t)}{dt^{n-1}} \quad (11)$$

$$B(s,t) = B_1(s)\frac{d\bar{u}(t)}{dt} + \cdots + B_{m-1}\frac{d^{m-1}\bar{u}(t)}{dt^{m-1}} \quad (12)$$

Because $\bar{u}(t)$ and $\bar{y}(t)$ are periodic after t_f, the integrals of the derivatives ($d^i\,\bar{y}(t)/dt^i$, $d^i\,\bar{u}(t)/dt^i$, i = 1,2,3,…) from t_f to $t_f + P_r$ are zero. So, Eqs. 13 and 14 are obtained, and Eq. 15 is obtained by integrating Eq. 6 from t_f to $t_f + P_r$

$$\int_{t_f}^{t_f+P_r}\left\{A(s,t) = A_1(s)\frac{d\bar{y}(t)}{dt} + \cdots + A_{n-1}\frac{d^{n-1}\bar{y}(t)}{dt^{n-1}}\right\}dt = 0 \quad (13)$$

$$\int_{t_f}^{t_f+P_r}\left\{B(s,t) = B_1(s)\frac{d\bar{u}(t)}{dt} + \cdots + B_{m-1}\frac{d^{m-1}\bar{u}(t)}{dt^{m-1}}\right\}dt = 0 \quad (14)$$

$$\int_{t_f}^{t_f+P_r}\bar{y}(t)dt = b_0\int_{t_f}^{t_f+P_r}\bar{u}(t)dt \quad (15)$$

Then, Eq. 16 is obtained by integrating Eq. 10 from t_f to $t_f + P_r$

$$G(s) = \frac{b_m s^m + \cdots + b_0}{a_n s^n + \cdots + 1} = \frac{s\int_{t_f}^{t_f+P_r} e^{st}\bar{y}_t(s)dt + \int_{t_f}^{t_f+P_r}\bar{y}(t)dt}{s\int_{t_f}^{t_f+P_r} e^{st}\bar{u}_t(s)dt + \int_{t_f}^{t_f+P_r}\bar{u}(t)dt} \quad (16)$$

In Eq. 16, $s\int_{t_f}^{t_f+P_r}\bar{y}_t(s)e^{st}dt$ can be rewritten to Eq. 17 by partial integration

$$\begin{aligned}&s\int_{t_f}^{t_f+P_r}\bar{y}_t(s)e^{st}dt = e^{st}\int_0^t e^{-s\tau}\bar{y}(\tau)d\tau\Big|_{t_f}^{t_f+P_r} - \int_{t_f}^{t_f+P_r}\bar{y}(t)dt\\ &= e^{st_f}\left((e^{sP_r}-1)\int_0^{t_f} e^{-s\tau}\bar{y}(\tau)d\tau + e^{sP_r}\int_{t_f}^{t_f+P_r} e^{-s\tau}\bar{y}(\tau)d\tau\right)\\ &\qquad - \int_{t_f}^{t_f+P_r}\bar{y}(t)dt\end{aligned} \quad (17)$$

Then, Algorithm 1 of Eq. 18 is derived from Eq. 16
Algorithm 1:

$$G(s) = \frac{(1-e^{-sP_r})\int_0^{t_f} e^{-s\tau}\bar{y}(\tau)d\tau + \int_{t_f}^{t_f+P_r} e^{-s\tau}\bar{y}(\tau)d\tau}{(1-e^{-sP_r})\int_0^{t_f} e^{-s\tau}\bar{u}(\tau)d\tau + \int_{t_f}^{t_f+P_r} e^{-s\tau}\bar{u}(\tau)d\tau} \quad (18)$$

By substituting $i\omega_j$ (j = 1,2,…,n) for s in Eq. 18, theoretically, the complete range of the frequency responses of the process can be obtained. However, as the period of P_r could be an extremely large value because it is the common multiple of P_i and P_f ($P_r = mP_i = nP_f$, m and n are integer), an excessively heavy computational burden is imposed by computing $\int_{t_f}^{t_f+P_r} e^{-s\tau}\bar{y}(\tau)d\tau$ and $\int_{t_f}^{t_f+P_r} e^{-s\tau}\bar{u}(\tau)d\tau$. Algorithm 1 is, therefore, not acceptable in practice.

Algorithm 2

Algorithm 2 is used to solve the practical problem of Algorithm 1, by replacing the integrals from t_f to $t_f + P_r$ in Algorithm 1 by integrals from t_f to $t_f + P_i$, and integrals from t_f to $t_f + P_f$. $\int_{t_f}^{t_f+P_r} e^{-s\tau}\bar{y}(\tau)d\tau$ in Eq. 18 is equivalent to Eq. 19

$$\int_{t_f}^{t_f+P_r} e^{-s\tau}\bar{y}(\tau)d\tau = \int_{t_f}^{t_f+P_r} e^{-s\tau}y(\tau)d\tau - \int_{t_f}^{t_f+P_r} e^{-s\tau}y_{ref}(\tau)d\tau \quad (19)$$

And, Eq. 20 is obtained from $P_r = nP_f$

$$\begin{aligned}\int_{t_f}^{t_f+P_r} e^{-s\tau}y(\tau)d\tau &= \int_{t_f}^{t_f+P_f} e^{-s\tau}y(\tau)d\tau\\ &+ \int_{t_f+P_f}^{t_f+2P_f} e^{-s\tau}y(\tau)d\tau + \ldots + \int_{t_f+(n-1)P_f}^{t_f+nP_f} e^{-s\tau}y(\tau)d\tau\end{aligned} \quad (20)$$

Since $y(\tau)$ is a periodic function, $y(\tau) = y\ (\tau - P_f)$ for $t_f + P_f \le \tau \le t_f + 2P_f$, $y(\tau) = y(\tau - 2P_f)$ for $t_f + 2P_f \le \tau \le t_f + 3P_f$,…. Then, Eq. 21 is easily derived from Eq. 20

$$\begin{aligned}&\int_{t_f}^{t_f+P_r} e^{-s\tau}y(\tau)d\tau\\ &= (1 + e^{-sP_f} + e^{-2sP_f} + \ldots + e^{-(n-1)sP_f})\int_{t_f}^{t_f+P_f} e^{-s\tau}y(\tau)d\tau\\ &= \frac{1-e^{-sP_f n}}{1-e^{-sP_f}}\int_{t_f}^{t_f+P_f} e^{-s\tau}y(\tau)d\tau\end{aligned} \quad (21)$$

The period of $y_{ref}(\tau)$ from t_f to $t_f + P_r$ is P_i and $P_r = mP_i$. So, Eq. 22 is obtained by the same procedure

$$\int_{t_f}^{t_f+P_r} e^{-s\tau} y_{ref}(\tau)d\tau = (1 + e^{-sP_i} + e^{-2sP_i} + \ldots + e^{-(m-1)sP_i})$$
$$\int_{t_f}^{t_f+P_i} e^{-s\tau} y_{ref}(\tau)d\tau = \frac{1 - e^{-sP_i m}}{1 - e^{-sP_i}} \int_{t_f}^{t_f+P_i} e^{-s\tau} y_{ref}(\tau)d\tau \quad (22)$$

Now, Eqs. 23 and 24 are straightforwardly obtained from $\bar{u} = u(t) - u_{ref}(t)$ and $\bar{y} = y(t) - y_{ref}(t)$

$$\int_{t_f}^{t_f+P_r} e^{-s\tau} \bar{u}(\tau)d\tau = \frac{1 - e^{-sP_f n}}{1 - e^{-sP_f}} \int_{t_f}^{t_f+P_f} e^{-s\tau} u(\tau)d\tau - \frac{1 - e^{-sP_i m}}{1 - e^{-sP_i}} \int_{t_f}^{t_f+P_i} e^{-s\tau} u_{ref}(\tau)d\tau \quad (23)$$

$$\int_{t_f}^{t_f+P_r} e^{-s\tau} \bar{y}(\tau)d\tau = \frac{1 - e^{-sP_f n}}{1 - e^{-sP_f}} \int_{t_f}^{t_f+P_f} e^{-s\tau} y(\tau)d\tau - \frac{1 - e^{-sP_i m}}{1 - e^{-sP_i}} \int_{t_f}^{t_f+P_i} e^{-s\tau} y_{ref}(\tau)d\tau \quad (24)$$

Then, Eq. 18 can be rewritten as Eq. 25

$$G(s) = \frac{\bar{a}_y + \frac{1-e^{-sP_f n}}{1-e^{-sP_f}} \int_{t_f}^{t_f+P_f} e^{-s\tau} y(\tau)d\tau - \frac{1-e^{-sP_i m}}{1-e^{-sP_i}} \int_{t_f}^{t_f+P_i} e^{-s\tau} y_{ref}(\tau)d\tau}{\bar{a}_u + \frac{1-e^{-sP_f n}}{1-e^{-sP_f}} \int_{t_f}^{t_f+P_f} e^{-s\tau} u(\tau)d\tau - \frac{1-e^{-sP_i m}}{1-e^{-sP_i}} \int_{t_f}^{t_f+P_i} e^{-s\tau} u_{ref}(\tau)d\tau} \quad (25)$$

where $\bar{a}_y = (1 - e^{-sP_r}) \int_0^{t_f} e^{-s\tau} \bar{y}(\tau)d\tau$ and $\bar{a}_u = (1 - e^{-sP_r}) \int_0^{t_f} e^{-s\tau} \bar{u}(\tau)d\tau$. By multiplying $(e^{-sP_r} - 1)(e^{-sP_i} - 1)$ to the numerator and denominator of Eq. 25, Eq. 26 is obtained

$$G(s) = \frac{\tilde{a}_y + (e^{-sP_f n} - 1)(e^{-sP_i} - 1) \int_{t_f}^{t_f+P_f} e^{-s\tau} y(\tau)d\tau - (e^{-sP_i m} - 1)(e^{-sP_f} - 1) \int_{t_f}^{t_f+P_i} e^{-s\tau} y_{ref}(\tau)d\tau}{\tilde{a}_u + (e^{-sP_f n} - 1)(e^{-sP_i} - 1) \int_{t_f}^{t_f+P_f} e^{-s\tau} u(\tau)d\tau - (e^{-sP_i m} - 1)(e^{-sP_f} - 1) \int_{t_f}^{t_f+P_i} e^{-s\tau} u_{ref}(\tau)d\tau} \quad (26)$$

where $\tilde{a}_y = (1 - e^{-sP_r})(e^{-sP_f} - 1)(e^{-sP_i} - 1) \int_0^{t_f} e^{-s\tau} \bar{y}(\tau)d\tau$ and $\tilde{a}_u = (1 - e^{-sP_r})(e^{-sP_f} - 1)(e^{-sP_i} - 1) \int_0^{t_f} e^{-s\tau} \bar{u}(\tau)d\tau$.

Without loss of generality, the period P_r, m and n can be assumed to be infinite. Then Eq. 27 is obtained

$$G(s) = \frac{\hat{a}_y - (e^{-sP_i} - 1) \int_{t_f}^{t_f+P_f} e^{-s\tau} y(\tau)d\tau + (e^{-sP_f} - 1) \int_{t_f}^{t_f+P_i} e^{-s\tau} y_{ref}(\tau)d\tau}{\hat{a}_u - (e^{-sP_i} - 1) \int_{t_f}^{t_f+P_f} e^{-s\tau} u(\tau)d\tau + (e^{-sP_f} - 1) \int_{t_f}^{t_f+P_i} e^{-s\tau} u_{ref}(\tau)d\tau} \quad (27)$$

where $\hat{a}_y = (e^{-sP_f} - 1)(e^{-sP_i} - 1) \int_0^{t_f} e^{-s\tau} \bar{y}(\tau)d\tau$, and $\hat{a}_u = (e^{-sP_f} - 1)(e^{-sP_i} - 1) \int_0^{t_f} e^{-s\tau} \bar{u}(\tau)d\tau$. By substituting $i\omega_j$ ($j = 1,2,\ldots,n$) for s in Eq. 27, Algorithm 2 of Eq. 28 is finally obtained

Algorithm 2:

$$G(i\omega_j) = \frac{a_y - (e^{-i\omega_j P_i} - 1) \int_{t_f}^{t_f+P_f} e^{-i\omega_j\tau} y(\tau)d\tau + (e^{-i\omega_j P_f} - 1) \int_{t_f}^{t_f+P_i} e^{-i\omega_j\tau} y_{ref}(\tau)d\tau}{a_u - (e^{-i\omega_j P_i} - 1) \int_{t_f}^{t_f+P_f} e^{-i\omega_j\tau} u(\tau)d\tau + (e^{-i\omega_j P_f} - 1) \int_{t_f}^{t_f+P_i} e^{-i\omega_j\tau} u_{ref}(\tau)d\tau}, \; j = 1, 2, \ldots, n \quad (28)$$

where $a_y = (e^{-i\omega_j P_f} - 1)(e^{-i\omega_j P_i} - 1) \int_0^{t_f} e^{-i\omega_j\tau} \bar{y}(\tau)d\tau$ and $a_u = (e^{-i\omega_j P_f} - 1)(e^{-i\omega_j P_i} - 1) \int_0^{t_f} e^{-i\omega_j\tau} \bar{u}(\tau)d\tau$.

The complete range of the frequency responses of the process for frequencies of ω_j, $j = 1,2,\ldots,n$ can be estimated by Algorithm 2 from the data of the activated process input and output of Figure 1c.

The proposed method presented as Algorithm 2 of Eq. 28 has the following advantages over the previous approaches. First, it can estimate theoretically the complete range of the frequency response data and the estimates are also exact. Second, it can be applied to the case of Figure 1c. No previous nonparametric estimation methods to estimate the complete range of the frequency responses can manipulate the case that the initial and final parts are both cyclic-steady-state. Third, it can be extended to all the cases of Figure 1a, b and c, which will be discussed in the next section. Fourth, it provides exact estimates under the environment of static disturbances. Assume that a static disturbance δ is added to the process input. As $\bar{u}(t) = (u(t) + \delta) - (u_{ref}(t) + \delta) = u(t) - u_{ref}(t)$ is not affected by the static disturbance, it is clear that Eq. 18 (equivalently, Eq. 28) provides the same estimates as those of the case with no static disturbance.

Algorithm 3

The proposed method for processes with initial cyclic-steady-state can be extended to the cases of Figure 1a in which the initial and final parts are both steady-state. For Figure 1a, the periods P_i and P_f can be chosen as the same infinitely small value because the initial and final parts are constant. That is, $P = P_i = P_f \rightarrow 0$ can be assumed. So, Eq. 27 can be rewritten to Eq. 29

$$G(s) = \lim_{P \rightarrow 0} \frac{(e^{-sP} - 1)^2 \int_0^{t_f} e^{-s\tau}\bar{y}(\tau)d\tau - (e^{-sP} - 1)\int_{t_f}^{t_f+P} e^{-s\tau}\bar{y}(\tau)d\tau}{(e^{-sP} - 1)^2 \int_0^{t_f} e^{-s\tau}\bar{u}(\tau)d\tau - (e^{-sP} - 1)\int_{t_f}^{t_f+P} e^{-s\tau}\bar{u}(\tau)d\tau} \tag{29}$$

After applying l'Hôpital's rule to Eq. 29, Eq. 30 is obtained

$$G(s) = \frac{s\int_0^{t_f} e^{-s\tau}\bar{y}(\tau)d\tau + e^{-st_f}\bar{y}(t_f)}{s\int_0^{t_f} e^{-s\tau}\bar{u}(\tau)d\tau + e^{-st_f}\bar{u}(t_f)} \tag{30}$$

Then, Eq. 31 is obtained by substituting $i\omega_j$ ($j = 1,2,\ldots,n$) for s in Eq. 30

Algorithm 3:

$$G(i\omega_j) = \frac{i\omega_j\int_0^{t_f} e^{-i\omega_j\tau}\bar{y}(\tau)d\tau + e^{-i\omega_j t_f}\bar{y}(t_f)}{i\omega_j\int_0^{t_f} e^{-i\omega_j\tau}\bar{u}(\tau)d\tau + e^{-i\omega_j t_f}\bar{u}(t_f)} \tag{31}$$

Therefore, Eq. 31 can estimate the complete range of the frequency responses of the process from the activated process data of which the initial and final parts are steady-state, as in the case of Figure 1a.

Algorithm 4

The proposed method of Eq. 28 can be extended to the case in which the initial part is zero-steady-state, and the final part is cyclic-steady-state as shown in Figure 1b. For Figure 1b, the period P_i can be assumed to be an infinitely small value because the initial part is constant; therefore, $P_i \rightarrow 0$ can be assumed. By applying l'Hôpital's rule to Eq. 27, Eq. 32 can be obtained

$$G(s) = \frac{s(e^{-sP_f} - 1)\int_0^{t_f} e^{-s\tau}\bar{y}(\tau)d\tau - s\int_{t_f}^{t_f+P_f} e^{-s\tau}y(\tau)d\tau - (e^{-sP_f} - 1)e^{-st_f}y_{ref}(t_f)}{s(e^{-sP_f} - 1)\int_0^{t_f} e^{-s\tau}\bar{u}(\tau)d\tau - s\int_{t_f}^{t_f+P_f} e^{-s\tau}u(\tau)d\tau - (e^{-sP_f} - 1)e^{-st_f}u_{ref}(t_f)} \tag{32}$$

By substituting $i\omega_j$ ($j = 1,2,\ldots,n$) for s in Eq. 32, Algorithm 4 of Eq. 33 is obtained

Algorithm 4:

$$G(i\omega_j) = \frac{(i\omega_j)(e^{-i\omega_j P_f} - 1)\int_0^{t_f} e^{-i\omega_j\tau}\bar{y}(\tau)d\tau - (i\omega_j)\int_{t_f}^{t_f+P_f} e^{-i\omega_j\tau}y(\tau)d\tau - (e^{-i\omega_j P_f} - 1)e^{-i\omega_j t_f}y_{ref}(t_f)}{(i\omega_j)(e^{-i\omega_j P_f} - 1)\int_0^{t_f} e^{-i\omega_j\tau}\bar{u}(\tau)d\tau - (i\omega_j)\int_{t_f}^{t_f+P_f} e^{-i\omega_j\tau}u(\tau)d\tau - (e^{-i\omega_j P_f} - 1)e^{-i\omega_j t_f}u_{ref}(t_f)} \tag{33}$$

Therefore, Eq. 33 can estimate the frequency responses of the process from the activated process data of which the initial part is steady-state and the final part is cyclic-steady-state, as in the case of Figure 1b.

Simulations

Consider the following third-order plus time delay process

$$G(s) = \frac{e^{-0.1s}}{(s+1)^3} \tag{34}$$

The process is activated by the proportional controller and relay feedback methods and the frequency response data are estimated by the proposed algorithms of Eqs. 28, 31 and 33 as shown in Figures 3–5. The results confirm the proposed algorithms' ability to provide the exact frequency response data of the process for all three cases. The describing function analysis for the initial part of Figure 1c provides significant estimation errors (ultimate frequency estimate: 1.50 (process: 1.54); ultimate gain estimate: 5.82 (process: 6.22)). Meanwhile, the proposed method provides exact estimates.

Figure 6a shows the activated process input and output when the measurements of the process output are contaminated by uniformly distributed random noises between −0.01 and 0.01. Figure 6b demonstrates the acceptable robustness to the measurement noises. Figure 7a is the case that an input static disturbance of 0.1 is entered in the beginning of the process activation. As expected, the proposed

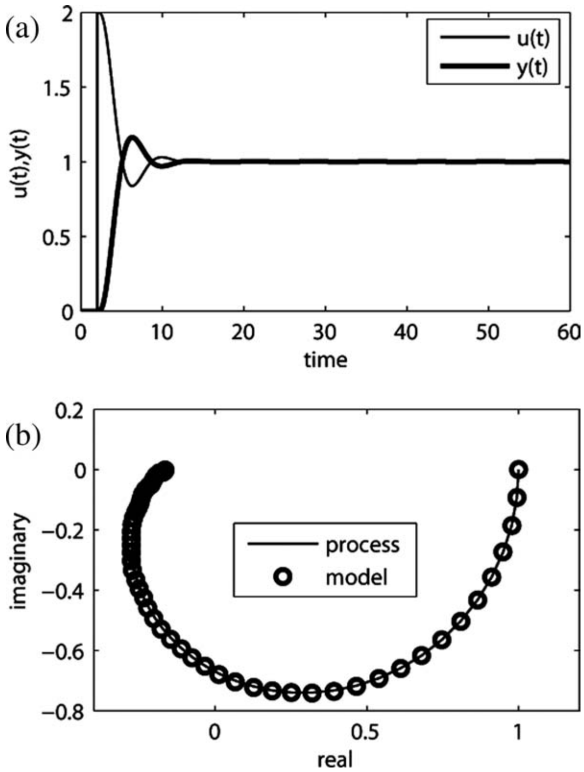

Figure 3. (a) Process activation (initial steady-state and final steady-state), and (b) identified frequency responses.

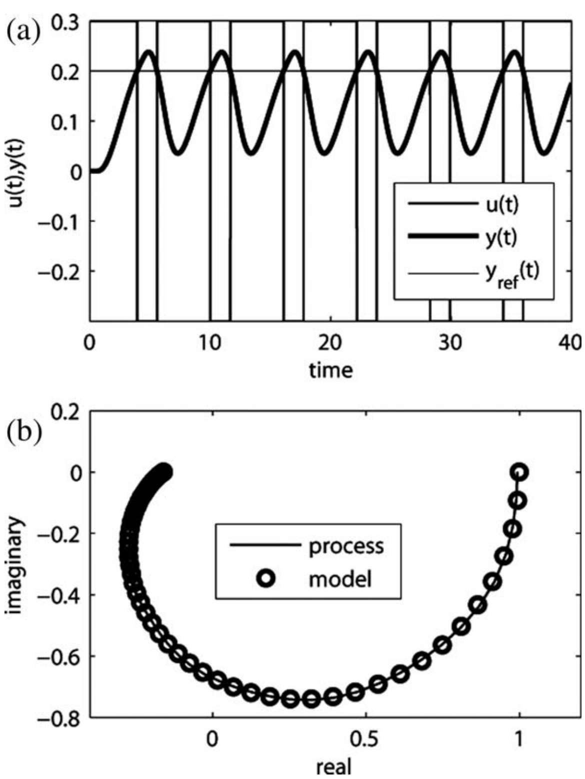

Figure 4. (a) Process activation (initial steady-state and final cyclic-steady-state), and (b) identified frequency responses.

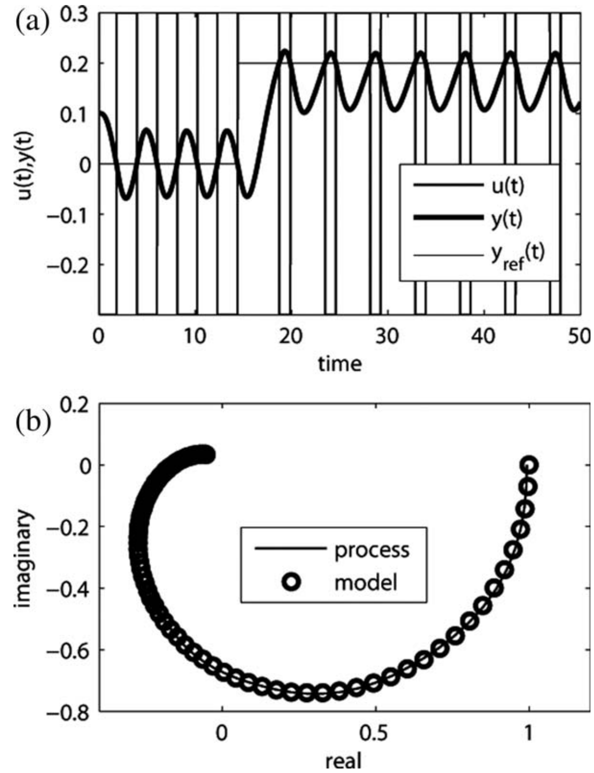

Figure 5. (a) Process activation (initial cyclic-steady-state and final cyclic-steady-state), and (b) identified frequency responses.

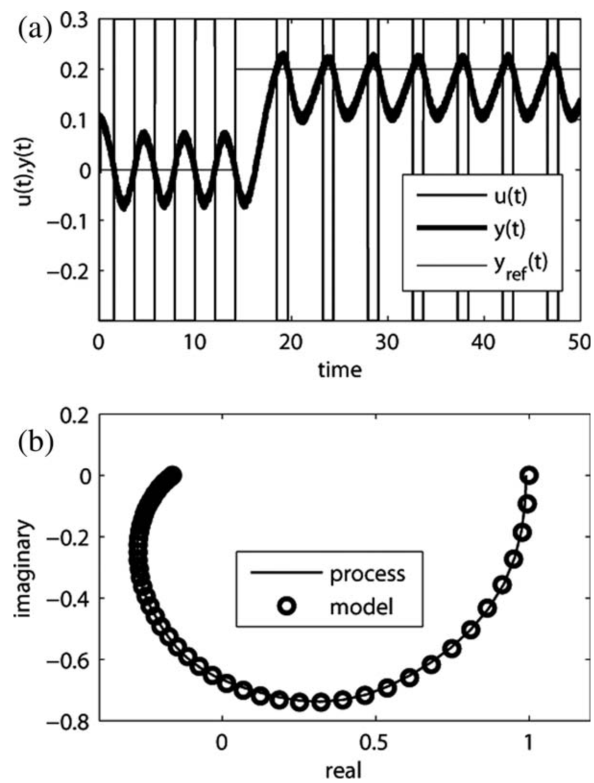

Figure 6. (a) Process activation in the case of measurement noises, and (b) identified frequency responses.

algorithm completely removes the effects of static input disturbances, and provides the exact frequency responses of the process as shown in Figure 7b.

Figure 8a shows the input and output data of the following process activated by the conventional biased-relay feedback method. The time constants of the process are widely spaced.

$$G(s) = \frac{e^{-0.1s}}{(10s+1)(s+1)(0.1s+1)} \tag{35}$$

Figure 8b confirms that the proposed method provides exact frequency responses for the process of Eq. 35.

Conclusions

A new process identification method is proposed to estimate a wide range of frequency responses of the process. The proposed method provides exact estimates and can incorporate all three cases of Figure 1. The simulation study confirmed the proposed method's ability to overcome the limitations of the previous approaches and completely remove the effects of static disturbances while maintaining robustness to measurement noises.

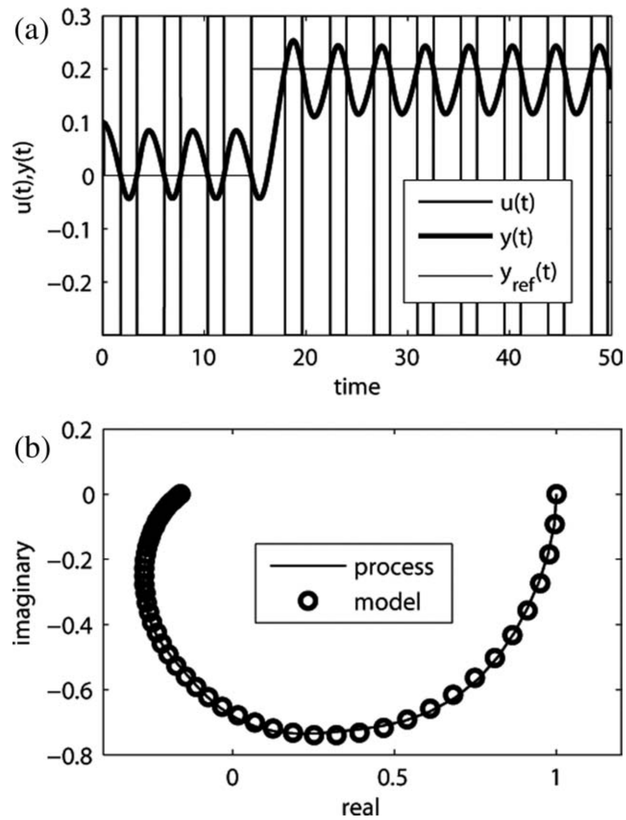

Figure 7. (a) Process activation in the case of a static input disturbance, and (b) identified frequency responses.

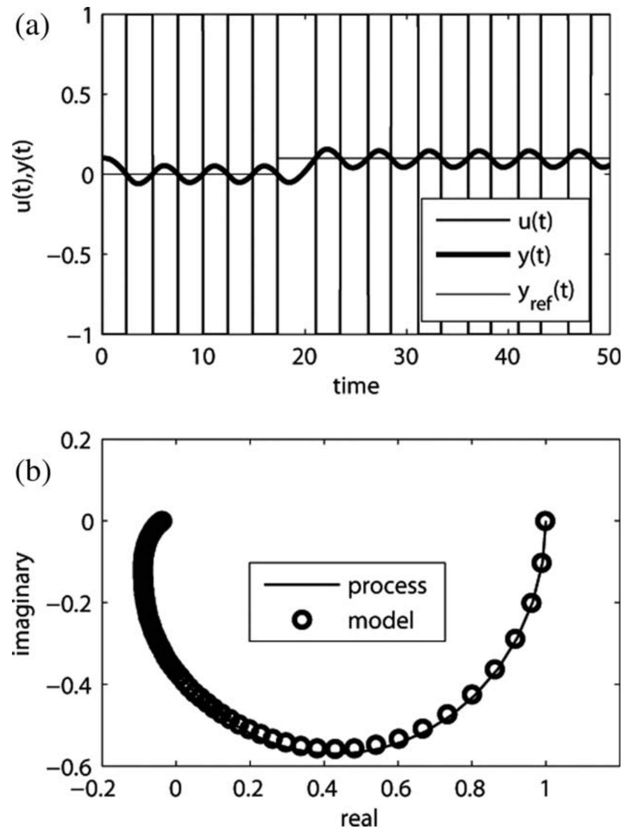

Figure 8. (a) Process activation in the case of widely spaced time constants, and (b) identified frequency responses.

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